

(a) Draw an electrical schematic (transistor-level circuit) for a 2-input NOR gate.

Solution: To draw an electrical schematic for a 2-input NOR gate:

If A or B is 1 $\rightarrow$ X is 0

If A and B are 0 $\rightarrow$ X is 1

A NOR B		X
0	0	1
0	1	0
1	0	0
1	1	0

Figure 1: Truth table for NOR gate

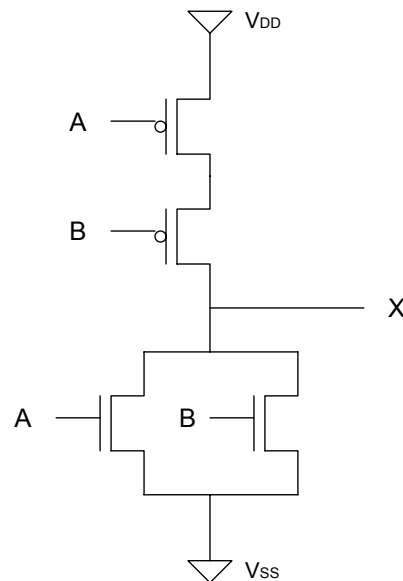


Figure 2: Electric schematic for NOR gate

(b) Draw a CMOS layout of a 2-input NOR gate.

Solution:

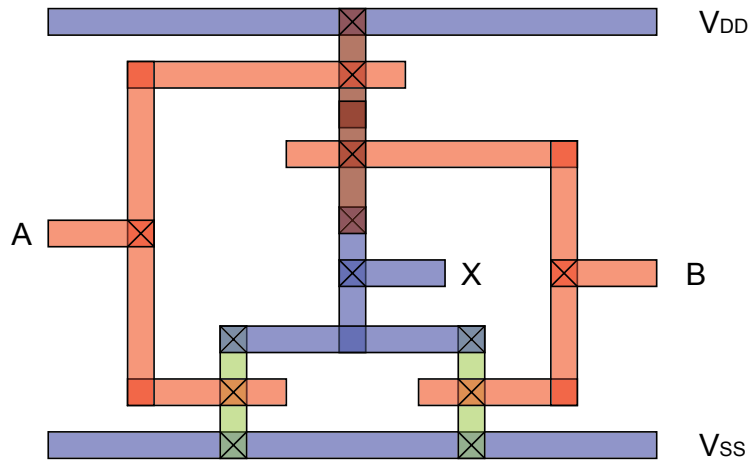


Figure 3: CMOS layout for NOR gate

(c) Draw an electrical schematic for a gate that computes $F = \overline{(A + B + C) \cdot D}$.

Solution: See Figure 4.

Problem 7-2. * Consider a bounded-degree routing network with n inputs and n outputs, where each input contains 1 packet. Show that if each of the n packets chooses an output destination randomly and independently, then the congestion is $\Omega(\lg n / \lg \lg n)$.

Solution: We will show that with very high probability there exists a destination to which $\Omega(\lg n / \lg \lg n)$ packets want to go. Since there are only $O(1)$ edges going into that destination, the congestion on at least one has to be $\Omega(\lg n / \lg \lg n)$. Since this happens whp, the expected congestion is also $\Omega(\lg n / \lg \lg n)$ (there's a lower bound of 1 in all cases).

Let $T = (1/2) \lg n / \lg \lg n$. Let E_i = the event that there are less than T packets that want to go to destination i . We want $\Pr[E_1 \cap \dots \cap E_n] \leq 2^{-n^{\Omega(1)}}$. We can write:

$$\begin{aligned}
 \Pr \left[\bigcap E_i \right] &= \Pr[E_1 \mid E_2 \cap \dots \cap E_n] \cdot \Pr[E_2 \cap \dots \cap E_n] \\
 &= \Pr[E_1 \mid E_2 \cap \dots \cap E_n] \cdot \Pr[E_2 \mid E_3 \cap \dots \cap E_n] \cdot \Pr[E_3 \cap \dots \cap E_n] \\
 &= \dots = \prod_{i=1}^n \Pr \left[E_i \mid \bigcap_{j>i} E_j \right] \\
 &\leq \left(1 - \frac{\Omega(1)}{T^T} \right)^n \quad (\text{proof below}) \\
 &= (1 - \Omega(1) \cdot 2^{-T \lg T})^n \leq \left(1 - 2^{-(1/2) \lg n} \right)^n \\
 &= \left[\left(1 - \frac{1}{\sqrt{n}} \right)^{\sqrt{n}} \right]^{\sqrt{n}} = \frac{1}{e} + o(1)^{\sqrt{n}} = 2^{-n^{\Omega(1)}}
 \end{aligned}$$

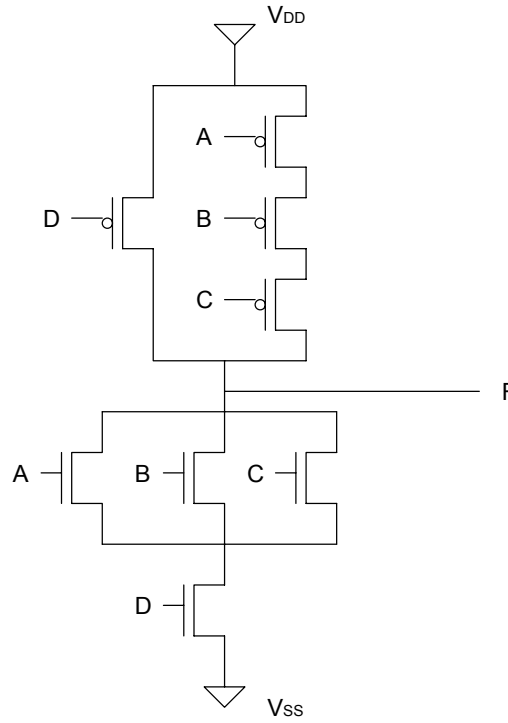


Figure 4: electrical schematic for a gate that computes $F = \overline{(A + B + C) \cdot D}$

It remains to show that $\Pr[E_i \mid \bigcap_{j>i} E_j] \leq 1 - \Omega(T^{-T})$. We analyze $\overline{E_i}$. This is the event that at least T packets want to go to destination i . This is at least the probability that exactly T packets want to go to destination i . There are $\binom{n}{T}$ choices for the T packets. The probability that one such T -subset of the packets decides to go to i , and the others decide to go somewhere else, is at least $\left(\frac{1}{n}\right)^T \left(1 - \frac{1}{n}\right)^{n-T}$. This is precisely the probability if we had no conditioning. Conditioning that not many elements go to other destinations can only increase the probability that more packets go to i . So our probability is:

$$\begin{aligned}
 \Pr[\overline{E_i} \mid \bigcap_{j>i} E_j] &\geq \binom{n}{T} \left(\frac{1}{n}\right)^T \left(1 - \frac{1}{n}\right)^{n-T} \\
 &\geq \binom{n}{T}^T \frac{1}{n^T} \left(1 - \frac{1}{n}\right)^n \\
 &\geq \frac{1}{T^T} \left(\frac{1}{e} + o(1)\right) = \Omega(T^{-T})
 \end{aligned}$$